Exercise 3

The midpoint of a piano string of tension T, density ρ, and length l is hit by a hammer whose head diameter is $2a$. A flea is sitting at a distance $l/4$ from one end. (Assume that $a < l/4$; otherwise, poor flea!) How long does it take for the disturbance to reach the flea?

Solution

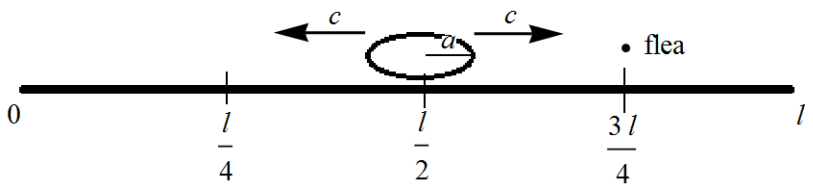

Figure 1: Schematic of the string and the flea.

When the hammer strikes the string, two waves start—one at $l/2 + a$ that travels to the right at speed c and one at $l/2 - a$ that travels to the left at speed c. For a string the speed is given by $c^2 = T/\rho$, and because the speed is constant, the formula we use is

$$\text{distance} = \text{rate} \cdot \text{time}$$
$$x = vt.$$

Because we are interested in the time, solve for t.

$$t = \frac{x}{v}$$
$$\text{time for wave to reach flea} = \frac{\text{distance from flea to where the wave starts}}{\text{wave speed}}$$
$$t = \frac{\frac{3l}{4} - \left(\frac{l}{2} + a\right)}{c} = \frac{\frac{l}{4} - a}{\sqrt{\frac{T}{\rho}}}$$

Therefore,

$$t = \left(\frac{l}{4} - a\right)\sqrt{\frac{\rho}{T}}.$$