

## Exercise 6

Prove that, among all possible dimensions, only in three dimensions can one have distortionless spherical wave propagation with attenuation. This means the following. A spherical wave in  $n$ -dimensional space satisfies the PDE

$$u_{tt} = c^2 \left( u_{rr} + \frac{n-1}{r} u_r \right),$$

where  $r$  is the spherical coordinate. Consider such a wave that has the special form  $u(r, t) = \alpha(r)f(r - \beta(r))$ , where  $\alpha(r)$  is called the attenuation and  $\beta(r)$  the delay. The question is whether such solutions exist for “arbitrary” functions  $f$ .

- (a) Plug the special form into the PDE to get an ODE for  $f$ .
- (b) Set the coefficients of  $f''$ ,  $f'$ , and  $f$  equal to zero.
- (c) Solve the ODEs to see that  $n = 1$  or  $n = 3$  (unless  $u \equiv 0$ ).
- (d) If  $n = 1$ , show that  $\alpha(r)$  is a constant (so that “there is no attenuation”).

(T. Morley, *American Mathematical Monthly*, Vol. 27, pp. 69–71, 1985)