

Exercise 4

Consider the diffusion equation $u_t = u_{xx}$ in $\{0 < x < 1, 0 < t < \infty\}$ with $u(0, t) = u(1, t) = 0$ and $u(x, 0) = 4x(1 - x)$.

- (a) Show that $0 < u(x, t) < 1$ for all $t > 0$ and $0 < x < 1$.
- (b) Show that $u(x, t) = u(1 - x, t)$ for all $t \geq 0$ and $0 \leq x \leq 1$.
- (c) Use the energy method to show that $\int_0^1 u^2 dx$ is a strictly decreasing function of t .