Exercise 7

(a) More generally, if $u_t - ku_{xx} = f$, $v_t - kv_{xx} = g$, $f \leq g$, and $u \leq v$ at $x = 0$, $x = l$ and $t = 0$, prove that $u \leq v$ for $0 \leq x \leq l$, $0 \leq t < \infty$.

(b) If $v_t - v_{xx} \geq \sin x$ for $0 \leq x \leq \pi$, $0 < t < \infty$, and if $v(0,t) \geq 0$, $v(\pi,t) \geq 0$ and $v(x,0) \geq \sin x$, use part (a) to show that $v(x,t) \geq (1 - e^{-t}) \sin x$.