Exercise 2

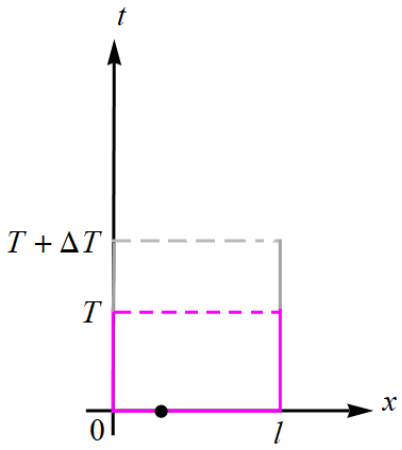
Consider a solution of the diffusion equation $u_t = u_{xx}$ in $\{0 \le x \le l,\ 0 \le t < \infty\}$.

(a) Let $M(T)$ = the maximum of $u(x,t)$ in the closed rectangle $\{0 \le x \le l,\ 0 \le t \le T\}$. Does $M(T)$ increase or decrease as a function of T?

(b) Let $m(T)$ = the minimum of $u(x,t)$ in the closed rectangle $\{0 \le x \le l,\ 0 \le t \le T\}$. Does $m(T)$ increase or decrease as a function of T?

Solution[1]

According to the maximum principle, the maximum value (or minimum value) of u occurs either initially or on the boundary. They can occur on the solid lines in the figure below.



Part (a)

Suppose that the maximum of u up until $t = T$ occurs at the black dot in the figure. If the values of u on the gray lines are less than that at the black dot, then the maximum will not change. The maximum will change, though, if u reaches an even higher value on the gray lines. That is, $M(T)$ increases as a function of T.

Part (b)

Suppose that the minimum of u up until $t = T$ occurs at the black dot in the figure. If the values of u on the gray lines are greater than that at the black dot, then the minimum will not change. The minimum will change, though, if u reaches an even lower value on the gray lines. That is, $m(T)$ decreases as a function of T.

[1] Thank you, J. V. Winkle, for the correction.