

Exercise 12

The purpose of this exercise is to calculate $Q(x, t)$ approximately for large t . Recall that $Q(x, t)$ is the temperature of an infinite rod that is initially at temperature 1 for $x > 0$, and 0 for $x < 0$.

- (a) Express $Q(x, t)$ in terms of $\mathcal{E}\text{rf}$.
- (b) Find the Taylor series of $\mathcal{E}\text{rf}(x)$ around $x = 0$. (*Hint:* Expand e^z , substitute $z = -y^2$, and integrate term by term.)
- (c) Use the first two nonzero terms in this Taylor expansion to find an approximate formula for $Q(x, t)$.
- (d) *Why* is this formula a good approximation for x fixed and t large?