

## Exercise 17

Solve the diffusion equation with variable dissipation:

$$u_t - ku_{xx} + bt^2u = 0 \quad \text{for } -\infty < x < \infty \quad \text{with } u(x, 0) = \phi(x),$$

where  $b > 0$  is a constant. (*Hint:* The solutions of the ODE  $w_t + bt^2w = 0$  are  $Ce^{-bt^3/3}$ . So make the change of variables  $u(x, t) = e^{-bt^3/3}v(x, t)$  and derive an equation for  $v$ .)