

Exercise 5

Let $f(x, t)$ be any function and let $u(x, t) = (1/2c) \iint_{\Delta} f$, where Δ is the triangle of dependence. Verify directly by differentiation that

$$u_{tt} = c^2 u_{xx} + f \quad \text{and} \quad u(x, 0) \equiv u_t(x, 0) \equiv 0.$$

(*Hint:* Begin by writing the formula as the *iterated* integral

$$u(x, t) = \frac{1}{2c} \int_0^t \int_{x-ct+cs}^{x+ct-cs} f(y, s) dy ds$$

and differentiate with care using the rule in the Appendix. This exercise is not easy.)