

Exercise 7

Let A be a positive-definite $n \times n$ matrix. Let

$$S(t) = \sum_{m=0}^{\infty} \frac{(-1)^m A^{2m} t^{2m+1}}{(2m+1)!}.$$

- (a) Show that this series of matrices converges uniformly for bounded t and its sum $S(t)$ solves the problem $S''(t) + A^2 S(t) = 0$, $S(0) = 0$, $S'(0) = I$, where I is the identity matrix. Therefore, it makes sense to denote $S(t)$ as $A^{-1} \sin tA$ and to denote its derivative $S'(t)$ as $\cos(tA)$.
- (b) Show that the solution of (13) is (14).