

### Exercise 13

Consider a string that is fixed at the end  $x = 0$  and is free at the end  $x = l$  except that a load (weight) of given mass is attached to the right end.

- (a) Show that it satisfies the problem

$$\begin{aligned}u_{tt} &= c^2 u_{xx} && \text{for } 0 < x < l \\u(0, t) &= 0 && u_{tt}(l, t) = -k u_x(l, t)\end{aligned}$$

for some constant  $k$ .

- (b) What is the eigenvalue problem in this case?
- (c) Find the equation for the positive eigenvalues and find the eigenfunctions.