

Exercise 9

On the interval $0 \leq x \leq 1$ of length one, consider the eigenvalue problem

$$\begin{aligned} -X'' &= \lambda X \\ X'(0) + X(0) &= 0 \quad \text{and} \quad X(1) = 0 \end{aligned}$$

(absorption at one end and zero at the other).

- (a) Find an eigenfunction with eigenvalue zero. Call it $X_0(x)$.
- (b) Find an equation for the positive eigenvalues $\lambda = \beta^2$.
- (c) Show graphically from part (b) that there are an infinite number of positive eigenvalues.
- (d) Is there a negative eigenvalue?