

Exercise 10

A string (of tension T and density ρ) with fixed ends at $x = 0$ and $x = l$ is hit by a hammer so that $u(x, 0) = 0$, and $\partial u / \partial t(x, 0) = V$ in $[-\delta + \frac{1}{2}l, \delta + \frac{1}{2}l]$ and $\partial u / \partial t(x, 0) = 0$ elsewhere. Find the solution explicitly in series form. Find the energy

$$E_n(h) = \frac{1}{2} \int_0^l \left[\rho \left(\frac{\partial h}{\partial t} \right)^2 + T \left(\frac{\partial h}{\partial x} \right)^2 \right] dx$$

of the n th harmonic $h = h_n$. Conclude that if δ is small (a concentrated blow), each of the first few overtones has almost as much energy as the fundamental. We could say that the tone is saturated with overtones.