

Exercise 8

A rod has length $l = 1$ and constant $k = 1$. Its temperature satisfies the heat equation. Its left end is held at temperature 0, its right end at temperature 1. Initially (at $t = 0$) the temperature is given by

$$\phi(x) = \begin{cases} \frac{5x}{2} & \text{for } 0 < x < \frac{2}{3} \\ 3 - 2x & \text{for } \frac{2}{3} < x < 1. \end{cases}$$

Find the solution, including the coefficients. (*Hint*: First find the equilibrium solution $U(x)$, and then solve the heat equation with initial condition $u(x, 0) = \phi(x) - U(x)$.)