

Exercise 5

Show that the Fourier sine series on $(0, l)$ can be derived from the full Fourier series on $(-l, l)$ as follows. Let $\phi(x)$ be any (continuous) function on $(0, l)$. Let $\tilde{\phi}(x)$ be its odd extension. Write the full series for $\tilde{\phi}(x)$ on $(-l, l)$. [Assume that its sum is $\tilde{\phi}(x)$.] By Exercise 4, this series has only sine terms. Simply restrict your attention to $0 < x < l$ to get the sine series for $\phi(x)$.