Exercise 15

Without any computation, predict which of the Fourier coefficients of \(|\sin x|\) on the interval \((-\pi, \pi)\) must vanish.

Solution

\(f(x) = |\sin x|\) is an even function because

\[ f(-x) = |\sin(-x)| = |-\sin x| = |\sin x| = f(x). \]

The Fourier series for \(|\sin x|\) on the interval \((-\pi, \pi)\) is

\[ |\sin x| = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos nx + B_n \sin nx), \]

where the coefficients are given by

\[ A_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{|\sin x| \cos nx}_{\text{even}} \, dx, \quad n = 0, 1, \ldots \]

\[ B_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{|\sin x| \sin nx}_{\text{odd}} \, dx = 0, \quad n = 1, 2, \ldots. \]

Since \(|\sin x| \sin nx\) is an odd function and it’s being integrated over a symmetric interval, the integral is equal to zero. Therefore, the coefficients of sine will vanish on the interval \((-\pi, \pi)\).