

Exercise 4

Consider the problem $u_t = ku_{xx}$ for $0 < x < l$, with the boundary conditions $u(0, t) = U$, $u_x(l, t) = 0$, and the initial condition $u(x, 0) = 0$, where U is a constant.

- (a) Find the solution in series form. (*Hint:* Consider $u(x, t) - U$.)
- (b) Using a direct argument, show that the series converges for $t > 0$.
- (c) If ϵ is a given margin of error, estimate how long a time is required for the value $u(l, t)$ at the endpoint to be approximated by the constant U within error ϵ . (*Hint:* It is an alternating series with first term U , so that the error is less than the next term.)