

Exercise 7

Show *by direct integration* that the eigenfunctions associated with the Robin BCs, namely,

$$\phi_n(x) = \cos \beta_n x + \frac{a_0}{\beta_n} \sin \beta_n x \quad \text{where } \lambda_n = \beta_n^2,$$

are mutually orthogonal on $0 \leq x \leq l$, where β_n are the positive roots of (4.3.8).