

## Exercise 12

Prove *Green's first identity*: For every pair of functions  $f(x)$ ,  $g(x)$  on  $(a, b)$ ,

$$\int_a^b f''(x)g(x) dx = - \int_a^b f'(x)g'(x) dx + f'g \Big|_a^b.$$

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### Solution

To solve this problem, one should use integration by parts. The formula for it is

$$\int_a^b u dv = uv \Big|_a^b - \int_a^b v du.$$

Starting from

$$\int_a^b f''(x)g(x) dx,$$

let

$$\begin{aligned} u &= g(x) & dv &= f''(x) dx \\ du &= g'(x) dx & v &= f'(x). \end{aligned}$$

Then we have

$$\int_a^b f''(x)g(x) dx = g(x)f'(x) \Big|_a^b - \int_a^b f'(x)g'(x) dx.$$

Therefore,

$$\int_a^b f''(x)g(x) dx = - \int_a^b f'(x)g'(x) dx + f'g \Big|_a^b.$$