

Exercise 4

- (a) Solve the problem $u_t = ku_{xx}$ for $0 < x < l$, $u(x, 0) = \phi(x)$, with the unusual boundary conditions

$$u_x(0, t) = u_x(l, t) = \frac{u(l, t) - u(0, t)}{l}.$$

Assume that there are no negative eigenvalues. (*Hint:* See Exercise 4.3.12.)

- (b) Show that as $t \rightarrow \infty$,

$$\lim u(x, t) = A + Bx,$$

assuming that you can take limits term by term.

- (c) Use Green's first identity and Exercise 3 to show that there are no negative eigenvalues.
- (d) Find A and B . (*Hint:* $A + Bx$ is the beginning of the series. Take the inner product of the series for $\phi(x)$ with each of the functions 1 and x . Make use of the orthogonality.)