

Exercise 10

Find the temperature of a metal rod that is in the shape of a solid circular cone with cross-sectional area $A(x) = b(1 - x/l)^2$ for $0 \leq x \leq l$, where b is a constant. Assume that the rod is made of a uniform material, is insulated on its sides, is maintained at zero temperature on its flat end ($x = 0$), and has an unspecified initial temperature distribution $\phi(x)$. Assume that the temperature is independent of y and z . [*Hint:* Derive the PDE $(1 - x/l)^2 u_t = k\{(1 - x/l)^2 u_x\}_x$. Separate variables $u = T(t)X(x)$ and then substitute $v(x) = (1 - x/l)X(x)$.]