Exercise 13

If friction is present, the wave equation takes the form

$$u_{tt} - c^2 u_{xx} = -r u_t,$$

where the resistance $r > 0$ is a constant. Consider a periodic source at one end: $u(0,t) = 0$, $u(l,t) = Ae^{i\omega t}$.

(a) Show that the PDE and the BC are satisfied by

$$\mathscr{U}(x,t) = Ae^{i\omega t}\frac{\sin \beta x}{\sin \beta l}, \quad \text{where } \beta^2 c^2 = \omega^2 - ir\omega.$$

(b) No matter what the IC, $u(x,0)$ and $u_t(x,0)$, are, show that $\mathscr{U}(x,t)$ is the asymptotic form of the solution $u(x,t)$ as $t \to \infty$.

(c) Show that you can get resonance as $r \to 0$ if $\omega = m\pi c/l$ for some integer m.

(d) Show that friction can prevent resonance from occuring.