

Exercise 3

Find the harmonic function $u(x, y)$ in the square $D = \{0 < x < \pi, 0 < y < \pi\}$ with the boundary conditions:

$$\begin{aligned} u_y = 0 & \quad \text{for } y = 0 \text{ and for } y = \pi, & u = 0 & \quad \text{for } x = 0 \text{ and} \\ & & u = \cos^2 y = \frac{1}{2}(1 + \cos 2y) & \quad \text{for } x = \pi. \end{aligned}$$