

Exercise 7

(*Rayleigh-Ritz approximation* to the harmonic function u in D with $u = h$ on bdy D .) Let w_0, w_1, \dots, w_n be arbitrary functions such that $w_0 = h$ on bdy D and $w_1 = \dots = w_n = 0$ on bdy D . The problem is to find constants c_1, \dots, c_n so that

$$w_0 + c_1 w_1 + \dots + c_n w_n \quad \text{has the least possible energy.}$$

Show that the constants must solve the linear system

$$\sum_{k=1}^n (\nabla w_j, \nabla w_k) c_k = -(\nabla w_0, \nabla w_j) \quad \text{for } j = 1, 2, \dots, n.$$