

**Exercise 7**

- (a) Solve the wave equation in three dimensions for  $t > 0$  with the initial conditions  $\phi(\mathbf{x}) = A$  for  $|\mathbf{x}| < \rho$ ,  $\phi(\mathbf{x}) = 0$  for  $|\mathbf{x}| > \rho$ , and  $\psi|_{\mathbf{x}} \equiv 0$ , where  $A$  is a constant. (This is somewhat like the plucked string.) (*Hint:* Differentiate the solution in Exercise 6(b).)
- (b) Sketch the regions in space-time that illustrate your answer. Where does the solution have jump discontinuities?
- (c) Let  $|\mathbf{x}_0| < \rho$ . Ride the wave along a light ray emanating from  $(\mathbf{x}_0, 0)$ . That is, look at  $u(\mathbf{x}_0 + t\mathbf{v}, t)$  where  $|\mathbf{v}| = c$ . Prove that

$$t \cdot u(\mathbf{x}_0 + t\mathbf{v}, t) \text{ converges as } t \rightarrow \infty.$$