Exercise 1

Let \( S \) be a characteristic surface for which \( S \cap \{ (x, y, z) : t = 0 \} \) is the sphere \( \{ x^2 + y^2 + z^2 = a^2 \} \). Describe \( S \) geometrically.

Solution

The sphere \( \{ x^2 + y^2 + z^2 = a^2 \} \) is represented in space-time as a circle in the \( xyz \)-plane.

In order to obtain the characteristic surface \( S \), draw light rays (with slope \( c \)) from every point on the circle.
$S$, highlighted in green in the figure below, is the union of the resulting surfaces and is completely hollow: it is a three-dimensional surface in four-dimensional space.

$$S = \{(x, y, z, t) \mid x^2 + y^2 + z^2 = (a \pm ct)^2\}$$

Taking a closer look at the origin, we note the following geometric facts about $S$. 

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