

Exercise 6

Show directly from the ODE (15) that the functions $H_k(x)e^{-x^2/2}$ are mutually orthogonal on the interval $(-\infty, \infty)$. That is

$$\int_{-\infty}^{\infty} H_k(x)H_l(x)e^{-x^2} dx = 0 \quad \text{for } k \neq l.$$

(*Hint:* See Section 5.3.)