Problem 1.25

Answer the same questions as in Problem 1.24, but for the differential equation $df/dt = -3f$.

Solution

$$\frac{df}{dt} = -3f$$

Divide both sides by f.

$$\frac{1}{f}\frac{df}{dt} = -3$$

The left side can be written as the derivative of a logarithm by the chain rule.

$$\frac{d}{dt}(\ln f) = -3$$

Integrate both sides with respect to t.

$$\ln f = -3t + C$$

Exponentiate both sides.

$$e^{\ln f} = e^{-3t+C}$$
$$f(t) = e^{-3t}e^{C}$$

Therefore, using a new constant A for e^C,

$$f(t) = Ae^{-3t}.$$

There's only one arbitrary constant here because this ODE is first-order. An alternative way to solve the ODE is by separating variables, Mr. Taylor's suggested method.

$$\frac{df}{dt} = -3f$$

$$\frac{df}{f} = -3\,dt$$

Integrate both sides.

$$\int \frac{df}{f} = \int -3\,dt$$
$$\ln f = -3t + C$$

Exponentiate both sides.

$$e^{\ln f} = e^{-3t+C}$$
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Therefore, using a new constant A for e^C,

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