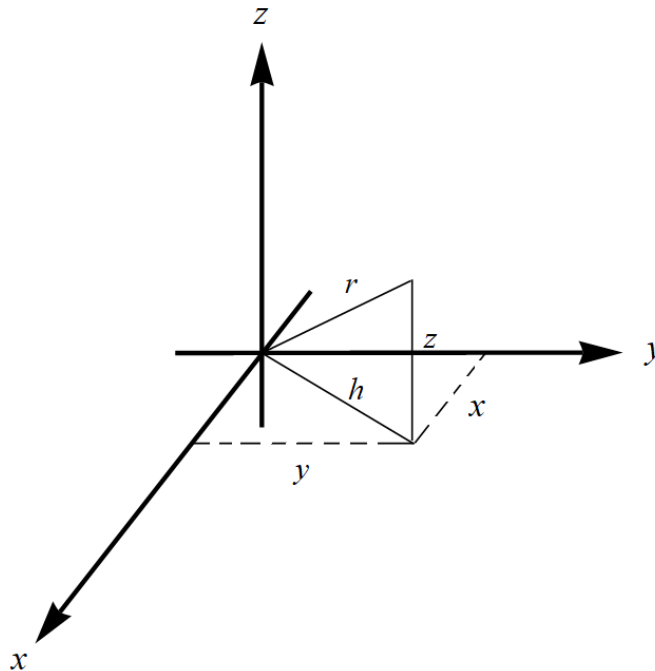


Problem 1.3

By applying Pythagoras's theorem (the usual two-dimensional version) twice over, prove that the length r of a three-dimensional vector $\mathbf{r} = (x, y, z)$ satisfies $r^2 = x^2 + y^2 + z^2$.

Solution



Apply the Pythagorean theorem to the triangle in the xy -plane.

$$x^2 + y^2 = h^2 \quad (1)$$

Apply the Pythagorean theorem again to the other triangle.

$$h^2 + z^2 = r^2$$

Substitute equation (1) for h^2 .

$$(x^2 + y^2) + z^2 = r^2$$

Therefore, the length of a three-dimensional vector $\mathbf{r} = (x, y, z)$ satisfies $r^2 = x^2 + y^2 + z^2$.