Problem 2.7

A flywheel weighing 70 lb was allowed to swing as a pendulum about a knife edge at the inner side of the rim, as shown in Fig. P2.7. If the measured period of oscillation was 1.22 s, determine the moment of inertia of the flywheel about its geometric axis.

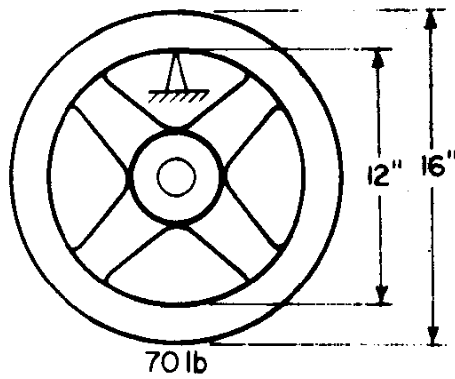
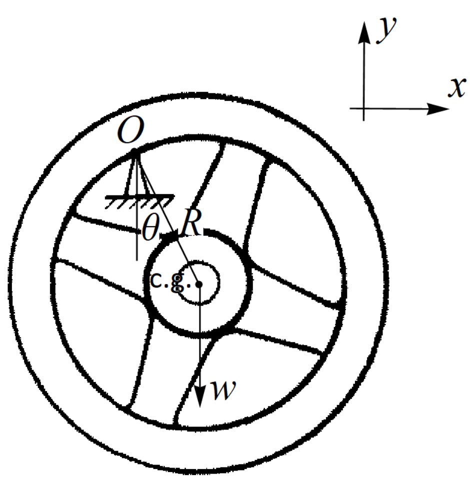


FIGURE P2.7.

Solution

If the flywheel is displaced some angle and allowed to oscillate, then it behaves as a physical pendulum.



Apply the rotational analog of Newton's second law in order to obtain the equation of motion.

$$\sum \boldsymbol{\tau} = J\boldsymbol{\alpha}$$

Here $\boldsymbol{\tau}$ is the external torque, J is the mass moment of inertia, and $\boldsymbol{\alpha}$ is the angular acceleration. Consider the sum of the torques about point O, the chosen origin.

$$\mathbf{r}_{\text{cg}} \times \mathbf{w} = J_O\boldsymbol{\alpha}$$

$$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ R\sin\theta & -R\cos\theta & 0 \\ 0 & -w & 0 \end{vmatrix} = J_O\alpha\hat{\mathbf{z}}$$

Evaluate the cross product.

$$-wR\sin\theta\hat{\mathbf{z}} = J_O\ddot{\theta}\hat{\mathbf{z}}$$

The components must then be equal.

$$-wR\sin\theta = J_O\ddot{\theta}$$

$$\ddot{\theta} = -\frac{wR}{J_O}\sin\theta$$

The motion is simple harmonic with the assumption that θ is small: $\sin\theta \approx \theta$.

$$\ddot{\theta} = -\frac{wR}{J_O}\theta$$

As a result, the angular frequency of oscillation is

$$\omega = \sqrt{\frac{wR}{J_O}}.$$

Write it in terms of the period τ and solve for J_O.

$$\frac{2\pi}{\tau} = \sqrt{\frac{wR}{J_O}}$$

$$J_O = \left(\frac{\tau}{2\pi}\right)^2 wR$$

Apply the parallel-axis theorem,

$$J_O = J_{\text{cg}} + \frac{w}{g}R^2,$$

in order to obtain the moment of inertia about a parallel axis through the flywheel's center of gravity.

$$J_{\text{cg}} + \frac{w}{g}R^2 = \left(\frac{\tau}{2\pi}\right)^2 wR$$

Therefore,

$$\begin{aligned} J_{\text{cg}} &= \left(\frac{\tau}{2\pi}\right)^2 wR - \frac{w}{g}R^2 \\ &= \left(\frac{1.22\text{ s}}{2\pi}\right)^2 (70\text{ lb})(6\text{ in}) - \frac{70\text{ lb}}{9.81\ \frac{\cancel{\text{m}}}{\text{s}^2} \times \frac{3.28\ \cancel{\text{ft}}}{1\ \cancel{\text{m}}} \times \frac{12\text{ in}}{1\ \cancel{\text{ft}}}}(6\text{ in})^2 \\ &\approx 9.31\text{ lb}\cdot\text{in}\cdot\text{s}^2. \end{aligned}$$