

Exercise 14

Find the closed form function for the following Taylor series:

$$f(x) = 2 + x - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots$$

[TYPO: “ $f(x) =$ ” is missing.]

Solution

$$\begin{aligned}f(x) &= 2 + x - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots \\f(x) &= 1 + x + \left(1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots\right) \\f(x) &= 1 + x + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}\end{aligned}$$

Therefore,

$$f(x) = 1 + x + \cos x.$$