

Exercise 2

Find the closed form function for the following Taylor series:

$$f(x) = 1 - 3x + \frac{9}{2}x^2 - \frac{9}{2}x^3 + \frac{27}{8}x^4 + \dots$$

Solution

$$f(x) = 1 - 3x + \frac{9}{2}x^2 - \frac{9}{2}x^3 + \frac{27}{8}x^4 + \dots$$

$$f(x) = 1 - \frac{(3x)^1}{1!} + \frac{(3x)^2}{2!} - \frac{(3x)^3}{3!} + \frac{(3x)^4}{4!} + \dots$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (3x)^n}{n!}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-3x)^n}{n!}$$

Therefore,

$$f(x) = e^{-3x}.$$