

Exercise 4

Find the closed form function for the following Taylor series:

$$f(x) = 1 - x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 - \frac{1}{5!}x^5 - \frac{1}{6!}x^6 + \dots$$

[**TYPO: This should be a minus sign.**]

Solution

In order to get the answer at the back of the book, the sign of the x^2 term should be negative.

$$\begin{aligned} f(x) &= 1 - x - \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 - \frac{1}{5!}x^5 - \frac{1}{6!}x^6 + \dots \\ f(x) &= \left(1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots\right) - \left(x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \dots\right) \\ f(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} - \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \end{aligned}$$

Therefore,

$$f(x) = \cos x - \sin x.$$