

Exercise 6

Find the closed form function for the following Taylor series:

$$f(x) = 2x + \frac{4}{3}x^3 + \frac{4}{15}x^5 + \frac{8}{315}x^7 + \dots$$

Solution

$$\begin{aligned} f(x) &= 2x + \frac{4}{3}x^3 + \frac{4}{15}x^5 + \frac{8}{315}x^7 + \dots \\ f(x) &= \frac{(2x)^1}{1!} + \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \frac{(2x)^7}{7!} + \dots \\ f(x) &= \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} (2x)^{2n+1} \end{aligned}$$

Therefore,

$$f(x) = \sinh 2x.$$