Exercise 2

Find the general solution for the following second order ODEs:

$$u'' - 2u' - 3u = 0$$

Solution

This is a homogeneous linear ODE with constant coefficients, so the solution is of the form, $u = e^{rx}$.

$$u = e^{rx} \quad \rightarrow \quad u' = re^{rx} \quad \rightarrow \quad u'' = r^2 e^{rx}$$

Substituting these into the equation gives us

$$r^2 e^{rx} - 2re^{rx} - 3e^{rx} = 0.$$

Divide both sides by e^{rx}.

$$r^2 - 2r - 3 = 0$$

Factor the left side.

$$(r - 3)(r + 1) = 0$$

$r = -1$ or $r = 3$. Therefore, the general solution is

$$u(x) = C_1 e^{-x} + C_2 e^{3x}.$$

We can check that this is the solution. The first and second derivatives are

$$\begin{aligned} u' &= -C_1 e^{-x} + 3C_2 e^{3x} \\ u'' &= C_1 e^{-x} + 9C_2 e^{3x}. \end{aligned}$$

Hence,

$$u'' - 2u' - 3u = \cancel{C_1 e^{-x}} + \cancel{9C_2 e^{3x}} - 2(\cancel{-C_1 e^{-x}} + \cancel{3C_2 e^{3x}}) - 3(\cancel{C_1 e^{-x}} + \cancel{C_2 e^{3x}}) = 0,$$

which means this is the correct solution.