Exercise 3

Find the general solution for the following second order ODEs:

$$u'' - u' - 2u = 0$$

Solution

This is a homogeneous linear ODE with constant coefficients, so the solution is of the form, $u = e^{rx}$.

$$u = e^{rx} \quad \to \quad u' = re^{rx} \quad \to \quad u'' = r^2 e^{rx}$$

Substituting these into the equation gives us

$$r^2 e^{rx} - re^{rx} - 2e^{rx} = 0.$$

Divide both sides by e^{rx}.

$$r^2 - r - 2 = 0$$

Factor the left side.

$$(r + 1)(r - 2) = 0$$

$r = -1$ or $r = 2$. Therefore, the general solution is

$$u(x) = C_1 e^{-x} + C_2 e^{2x}.$$

We can check that this is the solution. The first and second derivatives are

$$u' = -C_1 e^{-x} + 2C_2 e^{2x}$$
$$u'' = C_1 e^{-x} + 4C_2 e^{2x}.$$

Hence,

$$u'' - u' - 2u = \cancel{C_1 e^{-x}} + \cancel{4C_2 e^{2x}} - (\cancel{-C_1 e^{-x}} + \cancel{2C_2 e^{2x}}) - 2(\cancel{C_1 e^{-x}} + \cancel{C_2 e^{2x}}) = 0,$$

which means this is the correct solution.