Exercise 9

Find the general solution for the following initial value problems:

$$u'' - 3u' - 10u = 0, \quad u(0) = 2,\ u'(0) = 3$$

Solution

This is a homogeneous linear ODE with constant coefficients, so the solution is of the form, $u = e^{rx}$.

$$u = e^{rx} \quad \rightarrow \quad u' = re^{rx} \quad \rightarrow \quad u'' = r^2 e^{rx}$$

Substituting these into the equation gives us

$$r^2 e^{rx} - 3re^{rx} - 10e^{rx} = 0.$$

Divide both sides by e^{rx}.

$$r^2 - 3r - 10 = 0$$

Factor the left side.

$$(r-5)(r+2) = 0$$

$r = -2$ or $r = 5$, so the general solution is

$$u(x) = C_1 e^{-2x} + C_2 e^{5x}.$$

Because we have two initial conditions, we can determine C_1 and C_2.

$$u'(x) = -2C_1 e^{-2x} + 5C_2 e^{5x}$$

$$u(0) = C_1 e^0 + C_2 e^0 = 2$$
$$u'(0) = -2C_1 e^0 + 5C_2 e^0 = 3$$

Solving this system of equations gives $C_1 = 1$ and $C_2 = 1$. Therefore,

$$u(x) = e^{-2x} + e^{5x}.$$

We can check that this is the solution. The first and second derivatives are

$$u' = -2e^{-2x} + 5e^{5x}$$
$$u'' = 4e^{-2x} + 25e^{5x}.$$

Hence,

$$u'' - 3u' - 10u = \cancel{4e^{-2x}} + \cancel{25e^{5x}} - 3(\cancel{-2e^{-2x}} + \cancel{5e^{5x}}) - 10(\cancel{e^{-2x}} + \cancel{e^{5x}}) = 0,$$

which means this is the correct solution.