

Exercise 2

Find the sum of the following infinite series:

$$\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \cdots$$

Solution

Inspecting the series, we see that it is geometric. The first term is

$$a_1 = \frac{1}{2},$$

and the common ratio is

$$r = -\frac{1}{2}.$$

Therefore, the sum of the series is

$$\begin{aligned} S &= \frac{a_1}{1-r} \\ &= \frac{1/2}{3/2} \\ &= \frac{1}{3}. \end{aligned}$$