

Exercise 4

Find the sum of the following infinite series:

$$\frac{6}{7} \sin x + \frac{6}{49} \sin x + \frac{6}{343} \sin x + \frac{6}{2401} \sin x + \cdots$$

Solution

Inspecting the series, we see that it is geometric. The first term is

$$a_1 = \frac{6}{7} \sin x,$$

and the common ratio is

$$r = \frac{1}{7}.$$

Therefore, the sum of the series is

$$\begin{aligned} S &= \frac{a_1}{1 - r} \\ &= \frac{6/7}{6/7} \sin x \\ &= \sin x. \end{aligned}$$