

Exercise 12

For each of the following integral equations, classify as Fredholm, Volterra, or Volterra-Fredholm integral equation and find its kind. Classify the equation as singular or not.

$$u(x) = 1 + x^2 + \int_0^x \frac{1}{\sqrt{x-t}} u(t) dt$$

Solution

This is a Volterra integral equation because one of the limits of integration is not constant. It is of the second kind because u appears both inside and outside the integrals. It's inhomogeneous because of the $1 + x^2$. It is singular since the integrand becomes infinite at a point $t = x$ in the interval of integration. This equation in particular is known as an Abel integral equation since the exponent of $x - t$ in the denominator is $1/2$.