

## Exercise 10

Convert each of the following Volterra integral equation in 9–16 to an equivalent IVP:

$$u(x) = 1 + e^x - \int_0^x u(t) dt$$

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### Solution

Differentiate both sides with respect to  $x$ .

$$u'(x) = e^x - \frac{d}{dx} \int_0^x u(t) dt$$

$$u'(x) = e^x - u(x)$$

$$u' + u = e^x$$

The initial condition to this ODE is found by plugging in  $x = 0$  into the original integral equation.

$$u(0) = 1 + e^0 - \int_0^0 u(t) dt = 2$$

Therefore, the equivalent IVP is

$$u' + u = e^x, \quad u(0) = 2.$$