# Exercise 11

Convert each of the following Volterra integral equation in 9–16 to an equivalent IVP:

$$u(x) = 1 + x^2 + \int_0^x (x - t)u(t)\,dt$$

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### Solution

Differentiate both sides with respect to $x$.

$$u'(x) = 2x + \frac{d}{dx}\int_0^x (x - t)u(t)\,dt$$

Use the Leibniz rule to differentiate the integral.

$$u'(x) = 2x + \int_0^x \frac{\partial}{\partial x}(x - t)u(t)\,dt + (0)u(x)\cdot 1 - (x)u(0)\cdot 0$$

$$u' = 2x + \int_0^x u(t)\,dt$$

Differentiate both sides with respect to $x$ again.

$$u'' = 2 + \frac{d}{dx}\int_0^x u(t)\,dt$$

$$u'' = 2 + u(x)$$

$$u'' - u = 2$$

The initial conditions to this ODE are found by plugging in $x = 0$ into the original integral equation,

$$u(0) = 1 + 0^2 + \int_0^0 (0 - t)u(t)\,dt = 1,$$

and the formula for $u'$,

$$u'(0) = 2(0) + \int_0^0 u(t)\,dt = 0.$$

Therefore, the equivalent IVP is

$$u'' - u = 2,\ u(0) = 1,\ u'(0) = 0.$$