

Exercise 9

Convert each of the following Volterra integral equation in 9–16 to an equivalent IVP:

$$u(x) = x + 2 \int_0^x u(t) dt$$

Solution

Differentiate both sides with respect to x .

$$u'(x) = 1 + 2 \frac{d}{dx} \int_0^x u(t) dt$$

$$u'(x) = 1 + 2u(x)$$

$$u' - 2u = 1$$

The initial condition to this ODE is found by plugging in $x = 0$ into the original integral equation.

$$u(0) = 0 + 2 \int_0^0 u(t) dt = 0$$

Therefore, the equivalent IVP is

$$u' - 2u = 1, \quad u(0) = 0.$$