# Exercise 3

In Exercises 1–4, show that the given function $u(x)$ is a solution of the corresponding Fredholm integral equation:

$$u(x) = x + \int_{-1}^{1} (x^4 - t^4)u(t)\,dt,\ -1 \le x \le 1,\ u(x) = x$$

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### Solution

Substitute the function in question on both sides of the integral equation.

$$x \stackrel{?}{=} x + \int_{-1}^{1} (x^4 - t^4)t\,dt$$

Subtract $x$ from both sides.

$$\begin{aligned} 0 &\stackrel{?}{=} \int_{-1}^{1} (x^4 - t^4)t\,dt \\ &\stackrel{?}{=} \int_{-1}^{1} (x^4 t - t^5)\,dt \\ &\stackrel{?}{=} \int_{-1}^{1} x^4 t\,dt - \int_{-1}^{1} t^5\,dt \end{aligned}$$

$x^4$ doesn't depend on $t$, so it can be brought in front of the integral.

$$\begin{aligned} &\stackrel{?}{=} x^4 \int_{-1}^{1} t\,dt - \int_{-1}^{1} t^5\,dt \\ &\stackrel{?}{=} x^4 \cdot \left.\frac{t^2}{2}\right|_{-1}^{1} - \left.\frac{t^6}{6}\right|_{-1}^{1} \\ &\stackrel{?}{=} x^4 \left(\frac{1}{2} - \frac{1}{2}\right) - \left(\frac{1}{6} - \frac{1}{6}\right) \\ &= 0 \end{aligned}$$

Therefore,

$$u(x) = x$$

is a solution to the Fredholm integral equation.