Exercise 5

In Exercises 5–8, show that the given function $u(x)$ is a solution of the corresponding Volterra integral equation:

$$u(x) = 1 + \frac{1}{2}\int_0^x u(t)\,dt,\ u(x) = e^{2x}$$

[TYPO: The 1/2 should be a 2.]

Solution

Substitute the function in question on both sides of the integral equation.

$$e^{2x} \stackrel{?}{=} 1 + 2\int_0^x e^{2t}\,dt$$

Evaluate the integral.

$$\begin{aligned} e^{2x} &\stackrel{?}{=} 1 + 2 \cdot \frac{1}{2}e^{2t}\bigg|_0^x \\ &\stackrel{?}{=} 1 + (e^{2x} - e^0) \\ &= e^{2x} \end{aligned}$$

Therefore,

$$u(x) = e^{2x}$$

is a solution of the Volterra integral equation.