

## Problem 1.12

If the current flowing through an element is given by

$$i(t) = \begin{cases} 3t \text{ A}, & 0 \leq t < 6 \text{ s} \\ 18 \text{ A}, & 6 \leq t < 10 \text{ s} \\ -12 \text{ A}, & 10 \leq t < 15 \text{ s} \\ 0, & t \geq 15 \text{ s} \end{cases}$$

Plot the charge stored in the element over  $0 < t < 20 \text{ s}$ .

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### Solution

Current and charge are related by

$$i(t) = \frac{dq}{dt} \quad \rightarrow \quad q(t) = \int i(t) dt,$$

so the total charge is the area under the current-versus-time graph up until the given time.

$$q(t) = \begin{cases} \int_0^t 3s ds \text{ A}, & 0 \leq t < 6 \text{ s} \\ \int_0^6 3s ds \text{ A} + \int_6^t 18 ds \text{ A}, & 6 \leq t < 10 \text{ s} \\ \int_0^6 3s ds \text{ A} + \int_6^{10} 18 ds \text{ A} + \int_{10}^t (-12) ds \text{ A}, & 10 \leq t < 15 \text{ s} \\ \int_0^6 3s ds \text{ A} + \int_6^{10} 18 ds \text{ A} + \int_{10}^{15} (-12) ds \text{ A} + \int_{15}^t (0) ds \text{ A}, & t \geq 15 \text{ s} \end{cases}$$

$$= \begin{cases} \frac{3}{2}t^2 \text{ C}, & 0 \leq t < 6 \text{ s} \\ \frac{3}{2}(6)^2 \text{ C} + 18(t - 6) \text{ C}, & 6 \leq t < 10 \text{ s} \\ \frac{3}{2}(6)^2 \text{ C} + 18(10 - 6) \text{ C} + (-12)(t - 10) \text{ C}, & 10 \leq t < 15 \text{ s} \\ \frac{3}{2}(6)^2 \text{ C} + 18(10 - 6) \text{ C} + (-12)(15 - 10) \text{ C} + 0, & t \geq 15 \text{ s} \end{cases}$$

$$= \begin{cases} \frac{3}{2}t^2 \text{ C}, & 0 \leq t < 6 \text{ s} \\ (18t - 54) \text{ C}, & 6 \leq t < 10 \text{ s} \\ (-12t + 246) \text{ C}, & 10 \leq t < 15 \text{ s} \\ 66 \text{ C}, & t \geq 15 \text{ s} \end{cases}$$

Below is a plot of  $q(t)$  versus  $t$  for  $0 < t < 20$  s.

