Exercise 7.2.1

From Kirchhoff's law the current I in an RC (resistance-capacitance) circuit (Fig. 7.1) obeys the equation

$$R\frac{dI}{dt} + \frac{1}{C}I = 0.$$

(a) Find $I(t)$.

(b) For a capacitance of $10,000$ μF charged to 100 V and discharging through a resistance of 1 MΩ, find the current I for $t = 0$ and for $t = 100$ seconds.

Note. The initial voltage is I_0R or Q/C, where $Q = \int_0^\infty I(t)\,dt$.

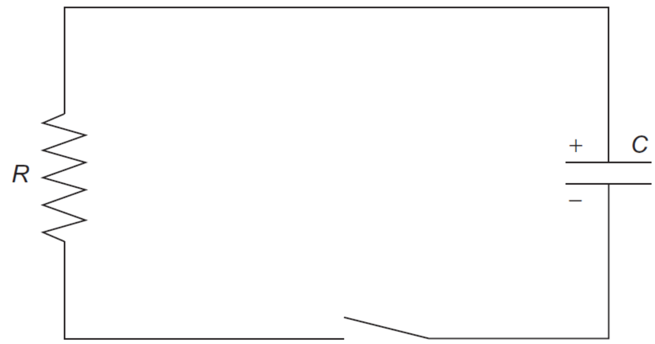

FIGURE 7.1 RC circuit.

Solution

Part (a)

$$RI' + \frac{1}{C}I = 0$$

Bring the second term to the right side.

$$RI' = -\frac{1}{C}I$$

Divide both sides by RI.

$$\frac{I'}{I} = -\frac{1}{RC}$$

The left side can be written as $d/dt(\ln I)$ by the chain rule.

$$\frac{d}{dt}(\ln I) = -\frac{1}{RC}$$

Integrate both sides with respect to t.

$$\ln I = -\frac{t}{RC} + C_1$$

Exponentiate both sides.

$$\begin{aligned} I(t) &= e^{-t/RC+C_1} \\ &= e^{C_1}e^{-t/RC} \end{aligned}$$

Use a new constant A for e^{C_1}.

$$I(t) = Ae^{-t/RC}$$

Part (b)

Use the relationship between voltage and charge for a capacitor to determine A.

$$\begin{aligned} V &= \frac{Q}{C} \\ &= \frac{1}{C}\int_0^\infty I(t)\,dt \\ &= \frac{1}{C}\int_0^\infty Ae^{-t/RC}\,dt \\ &= \frac{A}{C}(-RC)(0-1) \\ &= AR \end{aligned}$$

Consequently,

$$A = \frac{V}{R} = \frac{100 \text{ V}}{10^6\ \Omega} = 0.0001 \text{ amps}$$

and

$$I(t) = 0.0001 \exp\left(-\frac{t}{10000}\right).$$

Therefore,

$$\begin{aligned} I(0) &= 0.0001 \text{ amps} \\ I(100) &\approx 0.000099 \text{ amps}. \end{aligned}$$

The graph below shows $I(t)$ vs. t, the charge on the capacitor as a function of time.