

Exercise 24

Repeat Exercise 23 for the data given in Exercise 22. What do you observe?

Solution

The initial boundary value problem from Exercise 22 is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, \quad -\infty < t < \infty$$

$$u(x, 0) = \frac{1}{2} \sin 2\pi x + \frac{1}{4} \sin 4\pi x$$

$$\frac{\partial u}{\partial t}(x, 0) = 0$$

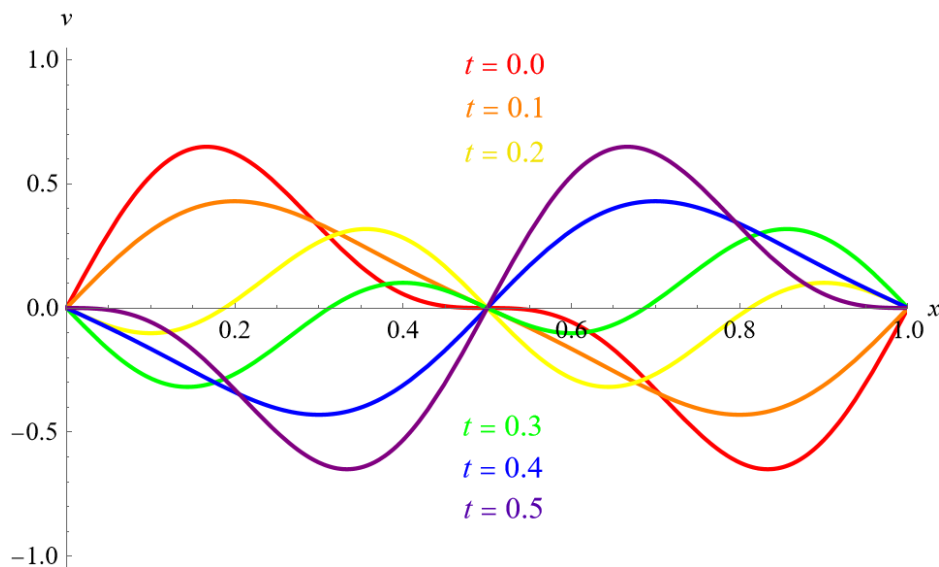
$$u(0, t) = 0$$

$$u(L, t) = 0,$$

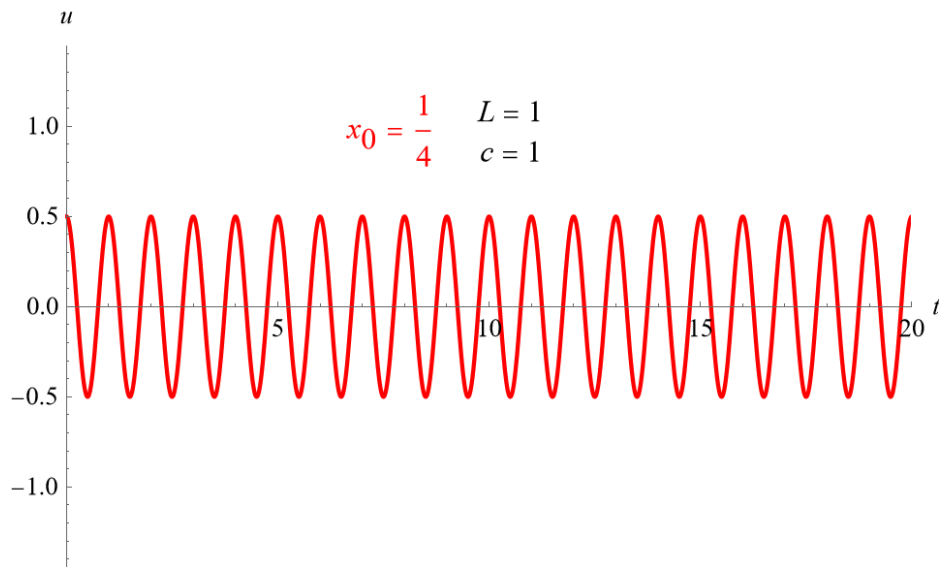
and its solution is

$$u(x, t) = \frac{1}{2} \sin \frac{2\pi x}{L} \cos \frac{2\pi ct}{L} + \frac{1}{4} \sin \frac{4\pi x}{L} \cos \frac{4\pi ct}{L}.$$

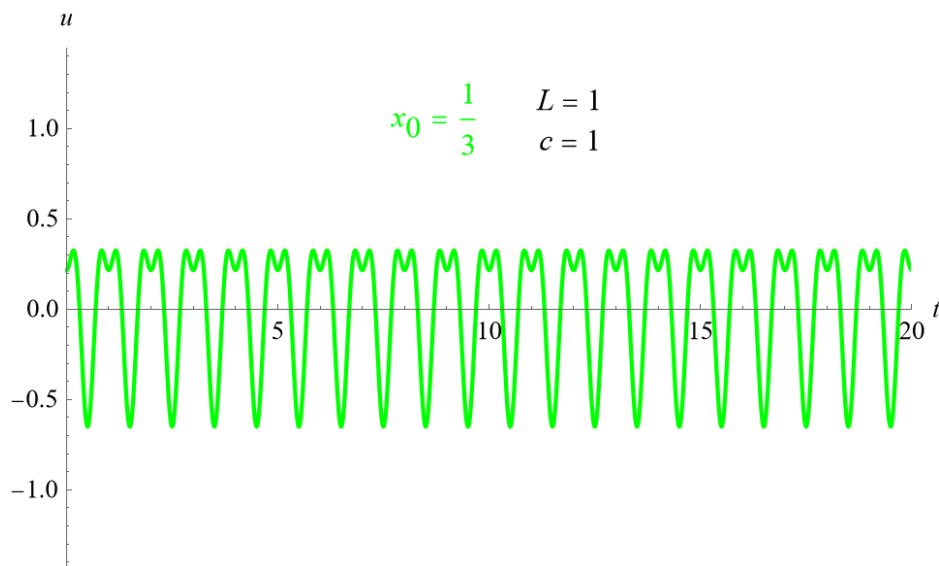
Below is a plot of u versus x over $0 < x < 1$ at several times with $c = 1$ and $L = 1$.



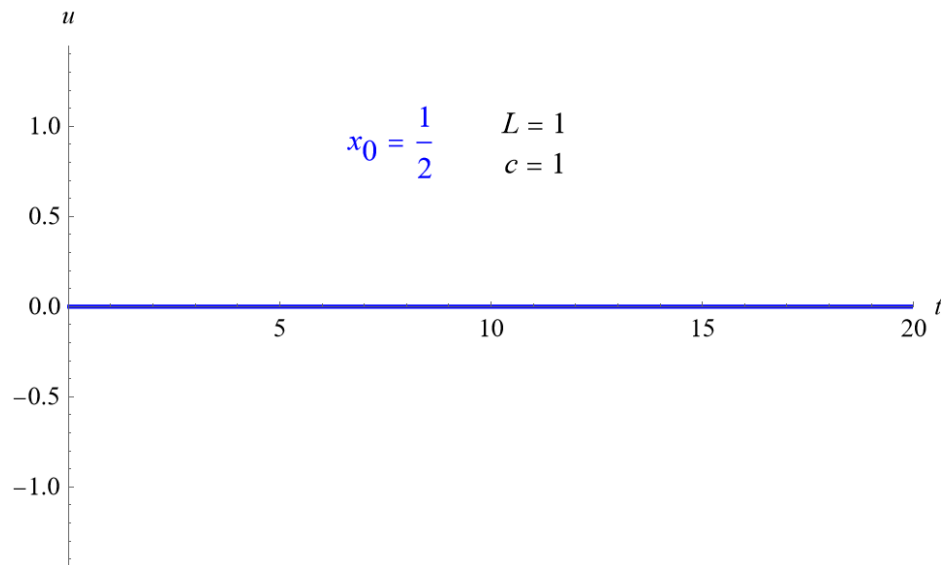
Below are plots of $u(x_0, t)$ versus t with $c = 1$ and $L = 1$ for $x_0 = \frac{1}{4}$,



for $x_0 = \frac{1}{3}$,



for $x_0 = \frac{1}{2}$,



and for $x_0 = \frac{3}{4}$.

