Problem 1A.3

Computation of the viscosities of gases at low density. Predict the viscosities of molecular oxygen, nitrogen, and methane at 20°C and atmospheric pressure, and express the results in $\text{mPa} \cdot \text{s}$. Compare the results with experimental data given in this chapter.

Answers: 0.0202, 0.0172, 0.0107 $\text{mPa} \cdot \text{s}$

Solution

The given temperature in Kelvin is $T = 20 + 273.15 = 293.15$ K, and the given pressure is $p = 1$ atm. Viscosity is calculated from $\mu = \mu_c \mu_r$, where μ_c is the critical viscosity and μ_r is the reduced viscosity.

Viscosity of Molecular Oxygen

From Table E.1 on page 864 we obtain the following facts about O_2.

$$T_c = 154.4 \text{ K} \qquad p_c = 49.7 \text{ atm} \qquad \mu_c = 250. \times 10^{-6} \, \frac{\text{g}}{\text{cm} \cdot \text{s}}$$

In order to determine μ_r, we have to know T_r and μ_r, the reduced temperature and reduced pressure, respectively.

$$T_r = \frac{T}{T_c} = \frac{293.15 \cancel{\text{K}}}{154.4 \cancel{\text{K}}} \approx 1.90$$

$$p_r = \frac{p}{p_c} = \frac{1 \cancel{\text{atm}}}{49.7 \cancel{\text{atm}}} \approx 0.02$$

Use the graph in Fig. 1.3-1 on page 22 to determine μ_r. Because of how small p_r is, we use the line representing the low-density limit.

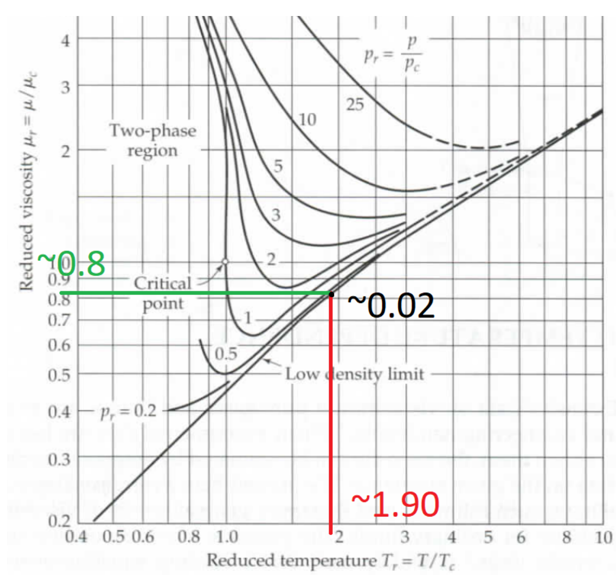

Figure 1: Use Fig. 1.3-1 in the text to determine μ_r.

We see that $\mu_r \approx 0.8$. Now we can find the viscosity.

$$\mu = \mu_c \mu_r \approx \left(250. \times 10^{-6} \ \frac{\text{g}}{\text{cm} \cdot \text{s}}\right) 0.8 = 2 \times 10^{-4} \ \frac{\text{g}}{\text{cm} \cdot \text{s}}$$

From Table F.3-4 on page 870, we get the conversion factor to Pa · s: 1 g/cm · s = 10^{-1} Pa · s.

$$\mu \approx 2 \times 10^{-4} \ \frac{\cancel{\text{g}}}{\cancel{\text{cm} \cdot \text{s}}} \times \frac{10^{-1} \ \cancel{\text{Pa} \cdot \text{s}}}{1 \ \cancel{\frac{\text{g}}{\text{cm} \cdot \text{s}}}} \times \frac{1000 \ \text{mPa} \cdot \text{s}}{1 \ \cancel{\text{Pa} \cdot \text{s}}}$$

Therefore, the viscosity of O_2 is

$$\mu \approx 0.02 \ \text{mPa} \cdot \text{s}.$$

The experimental value for the viscosity of O_2 is given in Table 1.1-3 on page 14: 0.0204 mPa · s. The percent difference between the two is

$$\text{Percent Difference} = \frac{0.02 - 0.0204}{0.0204} \times 100\% \approx -2\%.$$

Thus, the estimate here is below the experimental value by about 2%.

Viscosity of Molecular Nitrogen

From Table E.1 on page 864 we obtain the following facts about N_2.

$$T_c = 126.2 \ \text{K} \qquad p_c = 33.5 \ \text{atm} \qquad \mu_c = 180. \times 10^{-6} \ \frac{\text{g}}{\text{cm} \cdot \text{s}}$$

In order to determine μ_r, we have to know T_r and μ_r, the reduced temperature and reduced pressure, respectively.

$$T_r = \frac{T}{T_c} = \frac{293.15 \ \cancel{\text{K}}}{126.2 \ \cancel{\text{K}}} \approx 2.32$$
$$p_r = \frac{p}{p_c} = \frac{1 \ \cancel{\text{atm}}}{33.5 \ \cancel{\text{atm}}} \approx 0.03$$

Use the graph in Fig. 1.3-1 on page 22 to determine μ_r. Because of how small p_r is, we use the line representing the low-density limit.

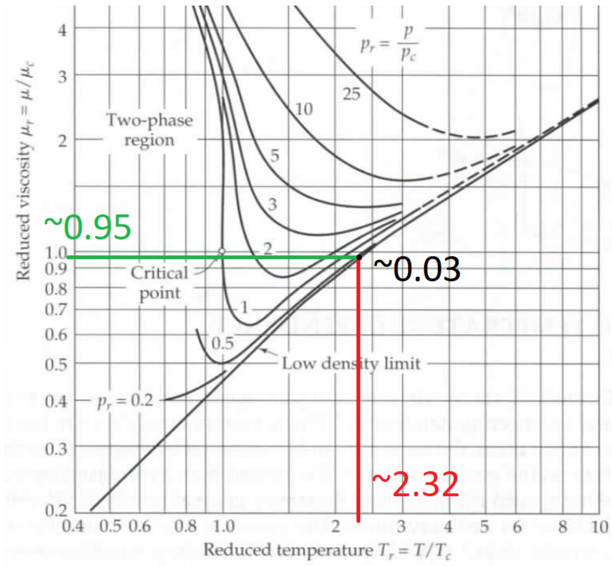

Figure 2: Use Fig. 1.3-1 in the text to determine μ_r.

We see that $\mu_r \approx 0.95$. Now we can find the viscosity.

$$\mu = \mu_c \mu_r \approx \left(180. \times 10^{-6} \frac{\text{g}}{\text{cm} \cdot \text{s}}\right) 0.95 \approx 1.7 \times 10^{-4} \frac{\text{g}}{\text{cm} \cdot \text{s}}$$

From Table F.3-4 on page 870, we get the conversion factor to Pa · s: 1 g/cm · s = 10^{-1} Pa · s.

$$\mu \approx 1.7 \times 10^{-4} \frac{\cancel{\text{g}}}{\cancel{\text{cm} \cdot \text{s}}} \times \frac{10^{-1} \cancel{\text{Pa} \cdot \text{s}}}{1 \frac{\cancel{\text{g}}}{\cancel{\text{cm} \cdot \text{s}}}} \times \frac{1000 \text{ mPa} \cdot \text{s}}{1 \cancel{\text{Pa} \cdot \text{s}}}$$

Therefore, the viscosity of N_2 is

$$\mu \approx 0.017 \text{ mPa} \cdot \text{s}.$$

The experimental value for the viscosity of N_2 is given in Table 1.1-3 on page 14: 0.0175 mPa · s. The percent difference between the two is

$$\text{Percent Difference} = \frac{0.017 - 0.0175}{0.0175} \times 100\% \approx -3\%.$$

Thus, the estimate here is below the experimental value by about 3%.

Viscosity of Methane

From Table E.1 on page 864 we obtain the following facts about CH_4.

$$T_c = 191.1 \text{ K} \qquad p_c = 45.8 \text{ atm} \qquad \mu_c = 159. \times 10^{-6} \frac{\text{g}}{\text{cm} \cdot \text{s}}$$

In order to determine μ_r, we have to know T_r and μ_r, the reduced temperature and reduced pressure, respectively.

$$T_r = \frac{T}{T_c} = \frac{293.15 \cancel{\text{K}}}{191.1 \cancel{\text{K}}} \approx 1.53$$

$$p_r = \frac{p}{p_c} = \frac{1 \cancel{\text{atm}}}{45.8 \cancel{\text{atm}}} \approx 0.02$$

Use the graph in Fig. 1.3-1 on page 22 to determine μ_r. Because of how small p_r is, we use the line representing the low-density limit.

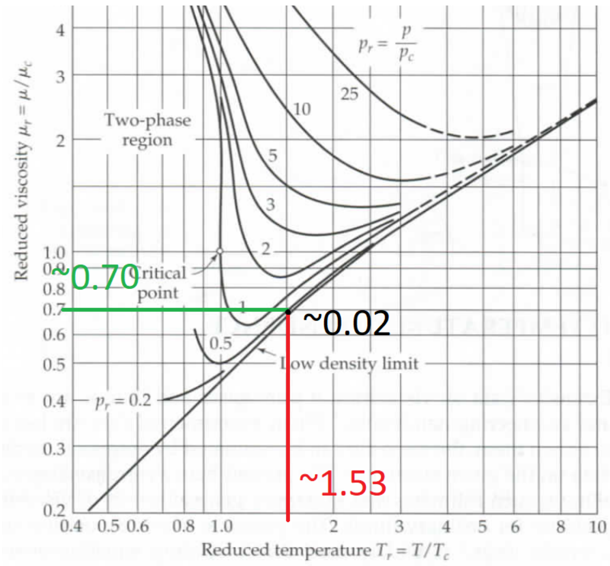

Figure 3: Use Fig. 1.3-1 in the text to determine μ_r.

We see that $\mu_r \approx 0.70$. Now we can find the viscosity.

$$\mu = \mu_c \mu_r \approx \left(159. \times 10^{-6} \frac{\text{g}}{\text{cm} \cdot \text{s}}\right) 0.70 \approx 1.1 \times 10^{-4} \frac{\text{g}}{\text{cm} \cdot \text{s}}$$

From Table F.3-4 on page 870, we get the conversion factor to Pa · s: 1 g/cm · s = 10^{-1} Pa · s.

$$\mu \approx 1.1 \times 10^{-4} \frac{\text{g}}{\text{cm} \cdot \text{s}} \times \frac{10^{-1} \text{ Pa} \cdot \text{s}}{1 \frac{\text{g}}{\text{cm} \cdot \text{s}}} \times \frac{1000 \text{ mPa} \cdot \text{s}}{1 \text{ Pa} \cdot \text{s}}$$

Therefore, the viscosity of CH_4 is

$$\mu \approx 0.011 \text{ mPa} \cdot \text{s}.$$

The experimental value for the viscosity of CH_4 is given in Table 1.1-3 on page 14: 0.0109 mPa · s. The percent difference between the two is

$$\text{Percent Difference} = \frac{0.011 - 0.0109}{0.0109} \times 100\% \approx 0.9\%.$$

Thus, the estimate here is above the experimental value by about 0.9%.