Problem 1A.5

Viscosities of chlorine-air mixtures at low density. Predict the viscosities (in cp) of chlorine-air mixtures at 75°F and 1 atm, for the following mole fractions of chlorine: 0.00, 0.25, 0.50, 0.75, 1.00. Consider air as a single component and use Eqs. 1.4-14 to 16.

Answers: 0.0183, 0.0164, 0.0150, 0.0139, 0.0130 cp

Solution

The strategy here is to determine the viscosities of Cl_2 and air alone using Eq. 1.4-14 and then to use them in Eqs. 1.4-15 and 16 to find the viscosity of the gas-mixture at various mole fractions of Cl_2. Eq. 1.4-14 gives the viscosity μ of a pure monatomic gas with molecular weight M in terms of the temperature T, the Lennard-Jones parameter σ, and the collision integral Ω_μ.

$$\mu = 2.6693 \times 10^{-5} \frac{\sqrt{MT}}{\sigma^2 \Omega_\mu} \tag{1.4-14}$$

Convert the given temperature into Kelvin.

$$T = \frac{5}{9}(75 + 459.67) \approx 297.04 \text{ K}$$

We obtain the following information about Cl_2 and air from Table E.1 on page 864. Note that κ is the Boltzmann constant.

	Cl_2	Air
Molecular Weight M:	70.905	28.964
Lennard-Jones Parameter σ:	4.115 Å	3.617 Å
Lennard-Jones Parameter ε/κ:	357 K	97.0 K

Hence, we have $\kappa T/\varepsilon \approx 297.04/357 \approx 0.83$ for Cl_2 and $\kappa T/\varepsilon \approx 297.04/97.0 \approx 3.06$ for air. Use Table E.2 on page 866 to look up the values of Ω_μ that correspond to these numbers. Use $\Omega_\mu = 1.762$ for Cl_2 and use $\Omega_\mu = 1.0344$ for air—these are averages of the values at 0.80 and 0.85 and 3.0 and 3.1, respectively. Now we can determine the viscosities.

$$\text{For } Cl_2, \quad \mu \approx 2.6693 \times 10^{-5} \frac{\sqrt{(70.905)(297.04)}}{(4.115)^2(1.762)} \approx 1.30 \times 10^{-4} \cancel{\text{poise}} \times \frac{100 \text{ cp}}{1 \cancel{\text{poise}}} = 0.0130 \text{ cp}$$

$$\text{For air}, \quad \mu \approx 2.6693 \times 10^{-5} \frac{\sqrt{(28.964)(297.04)}}{(3.617)^2(1.0344)} \approx 1.83 \times 10^{-4} \cancel{\text{poise}} \times \frac{100 \text{ cp}}{1 \cancel{\text{poise}}} = 0.0183 \text{ cp}$$

Our aim now is to find the viscosities of mixtures of Cl_2 and air by using Eqs. 1.4-15 and 16.

$$\mu_{\text{mix}} = \sum_{\alpha=1}^{N} \frac{x_\alpha \mu_\alpha}{\sum_{\beta=1}^{N} x_\beta \Phi_{\alpha\beta}}, \tag{1.4-15}$$

where $\Phi_{\alpha\beta}$ is

$$\Phi_{\alpha\beta} = \frac{1}{\sqrt{8}} \left(1 + \frac{M_\alpha}{M_\beta}\right)^{-1/2} \left[1 + \left(\frac{\mu_\alpha}{\mu_\beta}\right)^{1/2} \left(\frac{M_\beta}{M_\alpha}\right)^{1/4}\right]^2. \tag{1.4-16}$$

Since we have two chemical species, $N = 2$. Let Cl_2 be species 1 and let air be species 2, so $\mu_1 \approx 1.30 \times 10^{-4}$ poise and $\mu_2 \approx 1.83 \times 10^{-4}$ poise. Write out the terms of each sum in Eq. 1.4-15.

$$\begin{aligned}\mu_{\text{mix}} &= \sum_{\alpha=1}^{2} \frac{x_\alpha \mu_\alpha}{\sum_{\beta=1}^{2} x_\beta \Phi_{\alpha\beta}} \\ &= \frac{x_1\mu_1}{\sum_{\beta=1}^{2} x_\beta \Phi_{1\beta}} + \frac{x_2\mu_2}{\sum_{\beta=1}^{2} x_\beta \Phi_{2\beta}} \\ &= \frac{x_1\mu_1}{x_1\Phi_{11} + x_2\Phi_{12}} + \frac{x_2\mu_2}{x_1\Phi_{21} + x_2\Phi_{22}}\end{aligned}$$

Use Eq. 1.4-16 to calculate Φ_{11}, Φ_{12}, Φ_{21}, and Φ_{22}.

$$\Phi_{11} = \frac{1}{\sqrt{8}}\left(1 + \frac{M_1}{M_1}\right)^{-1/2}\left[1 + \left(\frac{\mu_1}{\mu_1}\right)^{1/2}\left(\frac{M_1}{M_1}\right)^{1/4}\right]^2 = 1$$

$$\Phi_{12} = \frac{1}{\sqrt{8}}\left(1 + \frac{M_1}{M_2}\right)^{-1/2}\left[1 + \left(\frac{\mu_1}{\mu_2}\right)^{1/2}\left(\frac{M_2}{M_1}\right)^{1/4}\right]^2 \approx 0.533220$$

$$\Phi_{21} = \frac{1}{\sqrt{8}}\left(1 + \frac{M_2}{M_1}\right)^{-1/2}\left[1 + \left(\frac{\mu_2}{\mu_1}\right)^{1/2}\left(\frac{M_1}{M_2}\right)^{1/4}\right]^2 \approx 1.83940$$

$$\Phi_{22} = \frac{1}{\sqrt{8}}\left(1 + \frac{M_2}{M_2}\right)^{-1/2}\left[1 + \left(\frac{\mu_2}{\mu_2}\right)^{1/2}\left(\frac{M_2}{M_2}\right)^{1/4}\right]^2 = 1$$

Plug all the numbers into the formula for μ_{mix} and then multiply the results by 100 to convert to centipoise (cp). Therefore,

Mole fraction of Cl_2 (x_1):	0.00	0.25	0.50	0.75	1.00
Mole fraction of air (x_2):	1.00	0.75	0.50	0.25	0.00 .
$\mu_{\text{mix}} \times 10^2$ (cp):	0.0183	0.0163	0.0149	0.0138	0.0130