Problem 1A.6

Estimation of liquid viscosity. Estimate the viscosity of saturated liquid water at 0°C and at 100°C by means of **(a)** Eq. 1.5-9, with $\Delta\hat{\mathcal{U}}_{\text{vap}} = 897.5$ Btu/lb$_m$ at 100°C, and **(b)** Eq. 1.5-11. Compare the results with the values in Table 1.1-2.

Answer: **(b)** 4.0 cp, 0.95 cp

Solution

Part (a)

Eq. 1.5-9 gives the viscosity μ of a liquid.

$$\mu = \frac{\tilde{N}h}{\tilde{V}} \exp(0.408\Delta\hat{\mathcal{U}}_{\text{vap}}/RT), \tag{1.5-9}$$

where $\tilde{N}$ is Avogadro's number (6.02214×10^{23} molecules/mol), h is Planck's constant (6.62608×10^{-34} J · s), $\tilde{V}$ is the molar volume, and $\Delta\hat{\mathcal{U}}_{\text{vap}}$ is the internal energy of vaporization at the normal boiling point. Consult a table on the thermodynamic properties of water to obtain the specific volume of saturated water at 0°C and 100°C. The molar volume can be obtained by multiplying it by the molar mass $2(1.008) + 16 = 18.016$ g/mol $= 0.018016$ kg/mol.

$$\begin{aligned} \text{At } 0^\circ\text{C}: \quad \tilde{V} &= 0.001000 \frac{\text{m}^3}{\cancel{\text{kg}}} \times \frac{0.018016 \cancel{\text{kg}}}{1 \text{ mol}} = 1.8016 \times 10^{-5} \frac{\text{m}^3}{\text{mol}} \\ \text{At } 100^\circ\text{C}: \quad \tilde{V} &= 0.001044 \frac{\text{m}^3}{\cancel{\text{kg}}} \times \frac{0.018016 \cancel{\text{kg}}}{1 \text{ mol}} = 1.88087 \times 10^{-5} \frac{\text{m}^3}{\text{mol}} \end{aligned}$$

Change $\Delta\hat{\mathcal{U}}_{\text{vap}}$ to SI units.

$$\Delta\hat{\mathcal{U}}_{\text{vap}} = 897.5 \frac{\cancel{\text{Btu}}}{\cancel{\text{lb}_m}} \times \frac{1.0550 \times 10^3 \text{ J}}{1 \cancel{\text{Btu}}} \times \frac{1 \cancel{\text{lb}_m}}{0.45359237 \cancel{\text{kg}}} \times \frac{0.018016 \cancel{\text{kg}}}{1 \text{ mol}} \approx 3.761 \times 10^4 \frac{\text{J}}{\text{mol}}$$

Use 8.31451 J/mol · K for R so that the units will cancel in the exponential function. In Kelvin, 0°C = 273.15 K and 100°C = 373.15 K. So we have

$$\begin{aligned} \text{At } 0^\circ\text{C}: \quad \mu &= \frac{(6.02214 \times 10^{23})(6.62608 \times 10^{-34})}{1.8016 \times 10^{-5}} \exp \frac{0.408(3.761 \times 10^4)}{(8.314)(273.15)} \approx 0.01903 \text{ Pa} \cdot \text{s} \\ \text{At } 100^\circ\text{C}: \quad \mu &= \frac{(6.02214 \times 10^{23})(6.62608 \times 10^{-34})}{1.88087 \times 10^{-5}} \exp \frac{0.408(3.761 \times 10^4)}{(8.314)(373.15)} \approx 0.002982 \text{ Pa} \cdot \text{s} \end{aligned}$$

Finally, convert the viscosities to units of centipoise (cp).

$$\begin{aligned} \text{At } 0^\circ\text{C}: \qquad & \mu \approx 0.01903 \cancel{\text{Pa} \cdot \text{s}} \times \frac{10^3 \text{ cp}}{1 \cancel{\text{Pa} \cdot \text{s}}} = 19.03 \text{ cp} \\ \text{At } 100^\circ\text{C}: \qquad & \mu \approx 0.002982 \cancel{\text{Pa} \cdot \text{s}} \times \frac{10^3 \text{ cp}}{1 \cancel{\text{Pa} \cdot \text{s}}} = 2.982 \text{ cp} \end{aligned}$$

Part (b)

Eq. 1.5-11 is the same as Eq. 1.5-9, but it uses Trouton's rule to estimate $\Delta\hat{\mathcal{U}}_{\text{vap}}$.

$$\mu = \frac{\tilde{N}h}{\tilde{V}} \exp(3.8T_b/T), \tag{1.5-11}$$

where T_b is the temperature of the boiling point. For water it is 373.15 K.

$$\text{At } 0°\text{C}: \quad \mu = \frac{(6.02214 \times 10^{23})(6.62608 \times 10^{-34})}{1.8016 \times 10^{-5}} \exp\frac{3.8(373.15)}{273.15} \approx 0.0040 \text{ Pa} \cdot \text{s}$$

$$\text{At } 100°\text{C}: \quad \mu = \frac{(6.02214 \times 10^{23})(6.62608 \times 10^{-34})}{1.88087 \times 10^{-5}} \exp\frac{3.8(373.15)}{373.15} \approx 0.00095 \text{ Pa} \cdot \text{s}$$

Convert the units to centipoise (cp).

$$\text{At } 0°\text{C}: \quad \mu \approx 0.0040 \, \cancel{\text{Pa} \cdot \text{s}} \times \frac{10^3 \text{ cp}}{1 \, \cancel{\text{Pa} \cdot \text{s}}} = 4.0 \text{ cp}$$

$$\text{At } 100°\text{C}: \quad \mu \approx 0.00095 \, \cancel{\text{Pa} \cdot \text{s}} \times \frac{10^3 \text{ cp}}{1 \, \cancel{\text{Pa} \cdot \text{s}}} = 0.95 \text{ cp}$$

Comparison of Results

The viscosity of water at 0°C and 100°C is given in Table 1.1-2: 1.787 mPa · s and 0.2821 mPa · s, respectively, or 1.787 cp and 0.2821 cp. For part (a) the percent difference is

$$\text{At } 0°\text{C}: \quad \text{Percent Difference} = \frac{19.03 - 1.787}{1.787} \times 100\% \approx 965\%$$

$$\text{At } 100°\text{C}: \quad \text{Percent Difference} = \frac{2.982 - 0.2821}{0.2821} \times 100\% \approx 957\% .$$

The numbers we calculated in part (a) are over 900% higher than the values in the table. We conclude that Eq. 1.5-9 is utterly useless for predicting the viscosity of water. For part (b) the percent difference is

$$\text{At } 0°\text{C}: \quad \text{Percent Difference} = \frac{4.0 - 1.787}{1.787} \times 100\% \approx 124\%$$

$$\text{At } 100°\text{C}: \quad \text{Percent Difference} = \frac{0.95 - 0.2821}{0.2821} \times 100\% \approx 237\%$$

The numbers we calculated in part (b) are over 100% higher than the values in the table. We conclude that Eq. 1.5-11 is utterly useless for predicting the viscosity of water.